

Time-Delay at Higher Genus in High-Energy Open String Scattering *

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abstract

We explore some aspects of causal time-delay in open string scattering studied recently by Seiberg, Susskind and Toumbas. By examining high-energy scattering amplitudes at higher order in perturbation theory, we argue that causal time-delay at G -th order is $1/(G+1)$ times smaller than the time-delay at tree level. We propose a space-time interpretation of the result by utilizing the picture of the high-energy open string scattering put forward by Gross and Mañes. We argue that the phenomenon of reduced time-delay is attributed to the universal feature of the space-time string trajectory in high-energy scattering that string shape at higher order remains the same as that at tree level but overall scale is reduced. We also discuss implications to the space-time uncertainty principle and make brief comments on causal time-delay behavior in space/time noncommutative field theory.

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1 Introduction

Noncommutative structures in string theory seem worth studying not only because they arise as a nonperturbative effect as in D-branes [1, 2, 3, 4, 5], Matrix theory [6, 7], or IIB matrix models [8, 9], but because they would reveal essential features of string theory as a unique theory known to date describing various extended objects (including strings themselves) in a consistent manner. However, recent progress in the study of these structures suggests that the extendedness of a string in the space-time cannot be described by a mere specification of noncommutativity relation such as

$$[X^\mu, X^\nu] = i\theta^{\mu\nu} \tag{1.1}$$

but requires more specification concerning microscopic details. In fact, quantum field theory on a noncommutative space defined by the above commutation relation with $\theta^{0i} \neq 0$, where i denotes one of the spatial directions, is found to behave as a theory consisting of extended objects like dipoles [10]. Consequently, from the viewpoint of pointlike quanta, the theory exhibits acausal behavior, nonlocality and nonunitarity [10, 11]. On the other hand, it is observed in [10, 12] that open strings living on the same noncommutative space do not show such pathologies and exhibit causal time-delays, reflecting their extendedness. Furthermore, a space/time noncommutative field theory cannot be obtained as the low-energy limit of string theory [13, 14, 15, 16]. Thus the open string exhibits another origin of its consistent extendedness other than noncommutativity described by (1.1). From this point of view, it is important to examine causal time-delay behavior of the open string theory in further details and check its consistency. Another importance is in that it seems related to the space-time uncertainty principle [17, 18]. This is as it should be, because the space-time uncertainty principle basically prescribes how a string extends in space-time in a way consistent with (perturbative) unitarity and analyticity. Therefore, studying the causal time-delay behavior may shed light on identifying central features in string theory dynamics.

In the present paper, we shall be exploring causal time-delay that show up in high-energy scattering of open strings at fixed order in string perturbation theory. Our motivation for the study comes from various sides. The high-energy scattering may enable us to explore the short-distance structure of string theory. The reason why we go beyond the tree-level analysis done in [10] is that in the high-energy scattering the contribution from higher genus to the amplitude is known to dominate the lower order ones [19, 20, 21]. Therefore, it is important to examine the causal time-delay at higher orders and see if it is pronounced similarly as compared to the result of [10]. Another motivation is that at high energy the space-time picture of the scattering becomes transparent because a certain class of classical trajectories become dominant [19, 20, 21]. This effect permits examining whether the interpretation of time-delays given in [10] persists to hold at higher genus as well.

The organization of this paper is as follows. In the next section, we give a brief recapitulation of the high-energy open string scattering at an arbitrary order in perturbation theory [21]. In section 3 we proceed to evaluate the causal time-delay at each order in perturbation theory and find that it is reduced gradually at higher orders. In section 4, we discuss space-time interpretation of our result using various features of the saddle point configuration [21]. Our analysis shows that the result is consistent with the interpretation in [10] based on the property that the string grows in the longitudinal direction with its energy. In the final section, we make brief comments on relation with the space-time uncertainty principle [17, 18] and comparison with the case of space/time noncommutative field theory.

2 High-Energy Scattering of Open Strings

In this section, for our foregoing analysis, we recapitulate relevant results concerning the high-energy scattering of open strings [21]. Consider, at genus G , four-point scattering amplitude in *closed* string theory given by:

$$\mathcal{A}_c^{(G)}(p_1, \dots, p_4) = g^{2G+2} \int \frac{\mathcal{D}g_{\alpha\beta}}{\mathcal{N}} \mathcal{D}X^\mu \exp \left(\frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu \right) \Pi_{i=1}^4 V_i(p_i), \quad (2.1)$$

where g is the closed string coupling constant, \mathcal{N} is the volume of the group of the diffeomorphisms and Weyl rescalings, and $V_i(p_i)$ are the vertex operators. In the case of the scattering of four tachyons, the Gaussian integration over X^μ yields

$$\mathcal{A}_c^{(G)}(p_i) = g^{2G+2} \int_{\mathcal{M}_G} [dm] \Pi_i d^2\xi_i \sqrt{g(\xi_i)} \Omega_c(m, \xi_i) \exp \left(-\alpha' \sum_{i < j} p_i \cdot p_j G_m(\xi_i, \xi_j) \right), \quad (2.2)$$

where m are a set of coordinates for the moduli space \mathcal{M}_G of genus G Riemann surfaces with four punctures, $\Omega_c(m, \xi_i)$ is the standard measure on \mathcal{M}_G , and G_m is the scalar Green function on the genus- G Riemann surface. In the kinematical limit all $p_i \cdot p_j$'s become large, one can perform the integration via saddle-point method and get [19, 20]

$$\mathcal{A}_c^{(G)}(p_i) \sim g^{2G+2} \Omega_c(\hat{m}, \hat{\xi}_i) (\mathcal{E}_c'')^{-\frac{1}{2}} \exp \left(-\alpha' \mathcal{E}_c(p_i, \hat{\xi}_i, \hat{m}_i) \right), \quad (2.3)$$

where

$$\mathcal{E}_c(p_i, \xi_i, m) \equiv \sum_{i < j} p_i \cdot p_j G_m(\xi_i, \xi_j) \quad (2.4)$$

is the electrostatic energy of an analog system of charges p_i at ξ_i on a Riemann surface with moduli m . $\hat{\xi}_i$ and \hat{m} denote the values at the saddle point where $(\mathcal{E}_c'')^{-1/2}$, the determinant of the second derivative of \mathcal{E}_c , is evaluated.

Following the strategy of [21], high-energy scattering amplitudes of open strings may be obtained from those of closed strings by utilizing the reflection principle: doubling an open

Riemann surface with boundaries yields a closed Riemann surface of higher genus. It turns out that, in the oriented open string theory, all relevant quantities associated with the original open Riemann surface can be extracted from the doubled, closed Riemann surface by imposing invariance under the reflection with respect to an appropriate symmetry plane [21]. Since (2.1) can be easily generalized to Riemann surfaces with boundaries, the above argument shows that, in the high energy scattering of four open strings as well, the amplitude are approximated by its saddle point expression at the G -th order in open string perturbation theory

$$\mathcal{A}_o^{(G)}(p_i) \sim g^{G+1} \Omega_o(\hat{m}, \hat{\xi}_i) (\mathcal{E}_o'')^{-\frac{1}{2}} \exp \left(-\alpha' \mathcal{E}_o(p_i, \hat{\xi}_i, \hat{m}_i) \right), \quad (2.5)$$

where Ω_o denotes the measure on the moduli space of open Riemann surfaces with boundaries,

$$\mathcal{E}_o(p_i, \xi_i, m) \equiv 2 \sum_{i < j} p_i \cdot p_j G_m(\xi_i, \xi_j), \quad (2.6)$$

is the analog electrostatic energy on the open Riemann surface, while G_m denotes the scalar Green function for the doubled, closed Riemann surface of genus G . In fact, it can be shown that every saddle-point configuration of the oriented open string can be constructed from the associated saddle-point configuration of the closed string via reflection principle [21]. The closed string saddle-point configuration at the G -th order was obtained in [19, 20] as the $(G+1)$ -sheeted Riemann surface of the form

$$y^{G+1} = \prod_{i=1}^4 (z - a_i)^{L_i}, \quad (2.7)$$

where L_i 's are relatively prime to $G+1$, $\sum_i L_i = 0 \pmod{G+1}$ [†] and the branch points a_i are separated by $1/(G+1)$ times the period. Therefore one can construct the corresponding open string saddle-points and also evaluate the electrostatic energy $\mathcal{E}_o^{(G)}$ on them. It is given by

$$\mathcal{E}_o^{(G)} = \frac{\mathcal{E}_o^{(0)}}{G+1}, \quad (2.8)$$

where

$$\mathcal{E}_o^{(0)} = s \ln |s| + t \ln |t| + u \ln |u|, \quad (2.9)$$

is the electrostatic energy on the disk, and s, t, u are the Mandelstam variables: $s = -(p_1 + p_2)^2$, $t = -(p_2 + p_3)^2$, $u = -(p_1 + p_3)^2$. Thus, the high-energy scattering amplitude of four open tachyons at the G -th order in perturbation theory is approximated by

$$\mathcal{A}_o^{(G)} \sim g^{G+1} \exp \left(-\alpha' \frac{s \ln |s| + t \ln |t| + u \ln |u|}{G+1} \right). \quad (2.10)$$

[†]Note that this condition prevents from considering all possible Riemann surfaces. Just for brevity, we will not treat the case of $L_i \neq 0 \pmod{G+1}$.

For later convenience, let us consider the nearly backward scattering $u \sim 0$ and compare the amplitude Eq.(2.10) at $G = 0$ with the Veneziano amplitude in the same kinematical region. The Veneziano amplitude is given by

$$\mathcal{A}_{st} \sim g \frac{\Gamma(-\alpha's)\Gamma(-\alpha't)}{\Gamma(1+\alpha'u)} + (t \leftrightarrow u) + (s \leftrightarrow u), \quad (2.11)$$

up to kinematical factors. For small u , the first term can be expanded as

$$\mathcal{A}_{st} \sim g \frac{\pi}{s \sin(\pi\alpha's)} (1 - \alpha'u \ln(\alpha's) + \dots), \quad (2.12)$$

where we use the identity

$$z\Gamma(-z)\Gamma(z) = \frac{-\pi}{\sin \pi z}. \quad (2.13)$$

It should be noted that as long as u is finite *and* non-zero, the second term in (2.12) is dominant in the high energy region, while setting u to zero, only the first term in (2.12) would survive. This manifests the fact that the high-energy limit $s \rightarrow \infty$ does not commute with the backward scattering limit $u \rightarrow 0$. Another way to see this is that the high-energy scattering amplitude (2.10) becomes trivial if one sets $u = 0$, a result which does not agree with (2.12). In fact, the above analysis cannot be applied to the exactly forward ($t = 0$) or exactly backward ($u = 0$) scattering, as, in such cases, the saddle point configurations approach boundaries of the moduli space \mathcal{M}_G , where two of the points ξ_i at which vertex operators are inserted become coincident.

To proceed further and examine causal time-delay, we will need to have at hand asymptotic scattering amplitudes which are valid even at higher perturbative order. Therefore, in the following, we shall consider the high-energy, fixed angle scattering with a moderate scattering angle $\phi \neq 0, \pi$, where we can rely on the Gross-Mañes amplitude (2.10). Note that this is in contrast to the case in [10] where the exactly backward scattering is considered and accordingly the scattering amplitude becomes singular in the sense that all poles contribute to it.

In concluding this section, it is worth pointing out a universal structure of the Gross-Mañes amplitude (2.10). At the G -th order in string perturbation theory, the G -dependence in (2.10) prompts to perform the following rescaling:

$$\alpha' \rightarrow \hat{\alpha}' \equiv \frac{\alpha'}{G+1}. \quad (2.14)$$

Defining rescaled kinematic invariants

$$\begin{aligned} \hat{s}\alpha' &\equiv s\hat{\alpha}' = \frac{s}{G+1}\alpha', \\ \hat{t}\alpha' &\equiv t\hat{\alpha}' = \frac{t}{G+1}\alpha', \end{aligned}$$

it follows that

$$\mathcal{A}_o^{(G)}(s, t) \sim g^G \mathcal{A}_o^{(0)}(\hat{s}, \hat{t}), \quad (2.15)$$

viz. the G -th order high-energy scattering amplitude can be reinterpreted as the tree-level amplitude at reduced values of kinematical invariants. As we will see later, universal features of the saddle point configurations yield a general and reasonable physical interpretation to the scaling (2.14).

3 Causal Time-Delay at Higher Genus

In this section, we would like to examine causal time-delay in the higher-order, high-energy scattering of four massless open string states. In order to do this, we first note that the saddle-point trajectory is universal and independent of the quantum numbers of scattered particles as the contribution from the vertex operators in (2.1) is at most the polynomials of momenta [19, 20, 21, 22]. Thus, up to kinematical prefactors, the tachyon amplitude (2.10) should be applicable to the massless case.

Let us consider the scattering of two massless open string states $1 + 2 \rightarrow 3 + 4$, closely following the analysis of [18]. For simplicity, we assume that the scattering takes place in a plane in a flat space-time and choose the center-of-mass system for the transverse momenta. As suggested in [18], one of the conceivable ways at present to extract the extendedness of strings from a scattering amplitude is that we first treat initial and final string states just as particle states by constructing their wave-packets with respect to the center-of-mass coordinates, and then observe the uncertainties of the interaction region, on which the extendedness would be reflected.

Following this method, we choose the wave-packet for each massless open string states as

$$\Phi_i(x_i, p_i) = \int d\mathbf{k}_i f_i(\mathbf{k}_i - \mathbf{p}_i) \exp(i(\mathbf{k}_i \cdot \mathbf{x}_i - |\mathbf{k}_i| t_i)), \quad (3.1)$$

where $f_i(\mathbf{k})$ is any function with a peak at $\mathbf{k} = 0$. Then the S-matrix element is given by

$$\langle 3, 4 | S | 1, 2 \rangle = \left(\prod_{i=1}^4 \int d\mathbf{k}_i \right) f_3^* f_4^* f_1 f_2 \delta\left(\sum_{i=1}^4 k_i\right) \mathcal{A}(s, t). \quad (3.2)$$

As proposed in [18], the uncertainty of the interaction region is measured by examining the response of the S-matrix under appropriate shifts of the particle trajectories in space-time. If we make shifts $t_i \rightarrow t_i + \Delta t_i$, then the wave-packet (3.1) becomes

$$\Phi_i(x_i, p_i; \Delta t_i) = \int d\mathbf{k}_i f_i(\mathbf{k}_i - \mathbf{p}_i) \exp(i(\mathbf{k}_i \cdot \mathbf{x}_i - |\mathbf{k}_i| t_i)) \exp(-i|\mathbf{k}_i| \Delta t_i). \quad (3.3)$$

For simplicity, set $\Delta t_1 = \Delta t_2 = -\Delta t_3 = -\Delta t_4 = \Delta t/2$ and $|\mathbf{k}_i| = E$ for all i , the integrand in (3.2) would acquire an additional phase factor $\exp(-2iE\Delta t)$ due to this shift. Then the

time-delay ΔT (uncertainty in time) can be estimated as the decay width of (3.2) with respect to $|\Delta t|$ under the insertion of this additional phase factor. As argued in [18], as Δt increases, the decay becomes appreciable when the variation of $\ln \mathcal{A}(E)$ is exceeded by the variation of the additional phase $2E\Delta t$. Therefore, we can set

$$\Delta T \sim \langle |\Delta t| \rangle \sim \frac{1}{2} \left| \frac{\partial}{\partial E} \ln \mathcal{A}(E) \right|. \quad (3.4)$$

Substituting the Gross-Mañes amplitude (2.10) into this equation and setting $E = |\mathbf{p}| = p_0$, we immediately obtain

$$\Delta T \sim \frac{\alpha' p_0}{G+1} f(\phi), \quad (3.5)$$

where $f(\phi)$ is defined as

$$f(\phi) \equiv -\sin^2 \frac{\phi}{2} \ln \sin^2 \frac{\phi}{2} - \cos^2 \frac{\phi}{2} \ln \cos^2 \frac{\phi}{2}, \quad (3.6)$$

thus fixed in our kinematical region. Although we cannot determine the sign of the time uncertainty by this method, we conclude that this is causal, because at tree level it is argued in [10, 15] that the string scattering is indeed causal, and even at higher genus, the scattering process remains almost the same as that at tree level due to universal features of the Gross-Mañes saddle point, as we will discuss in the next section. The reduced causal time-delay (3.5) is the main result in this section. It displays the fact that the causal time-delay at G -th order in string perturbation theory is reduced by $1/(G+1)$ compared to that at tree level. Here we note that, in (3.4), a numerical constant we have omitted should be independent of the genus, because it originates only from kinematics of the scattering process. Rather, it may be possible to take (3.4) as one of definitions of ΔT in the framework here.

It should be emphasized that the time-delay at each order in perturbation theory is purely theoretical: what we will observe here must be a time-delay associated with the full amplitude. Nevertheless we believe our reduced time delay is important because, as we discussed later, it provides reasonable physical pictures and important implications of high-energy behavior of strings in the framework of perturbative string theory.

4 Reduction of Time-Delay: Space-Time Interpretation

In this section, we would like to draw certain space-time interpretation concerning the higher-genus reduction of the causal time-delay (3.5) by collecting various features the saddle-point exhibits.

Let us begin with the saddle-point trajectory of the closed string as originally found in [20]. It is given by

$$X^\mu(z) = \frac{i}{G+1} \sum_{i=1}^4 \alpha' p_i^\mu \ln |z - a_i| + \mathcal{O}(1/s), \quad (4.1)$$

where z denotes the point on the $(G+1)$ -sheeted Riemann surface (2.7). It is worth noting that, from (4.1), even at higher order, the shape of the saddle point trajectory remains the same as that at tree level, except that the overall *space-time* scale is reduced by a factor $1/(G+1)$. By examining the behavior at the vicinity of the branch points a_i , where the four vertex operators are inserted, one can easily get the following space-time picture of the saddle point trajectory [20]: each of the two incoming closed strings winds around a closed curve $(G+1)$ times, then they interact and propagate as $(G+1)$ many intermediate short closed strings.[‡] Subsequently, the $G+1$ short strings then rejoin together and finally produce two separate $(G+1)$ -times wound outgoing closed strings. This picture is consistent with the fact that, as shown in (4.1), the string trajectory at G -th order is scaled down by a factor $1/(G+1)$ compared to that at tree level.

As recapitulated in the previous section, any oriented open string diagram can be obtained from corresponding closed string diagram by cutting each closed string into two open strings and keep one side of them. Therefore, high-energy scattering amplitude and space-time picture for open string can be obtained, via reflection principle, from those for closed string. From the space-time picture of the open string trajectory deduced this way, it follows immediately that, for the open strings as well, the space-time trajectory of the saddle point configuration at higher order in perturbation theory is exactly the same as that at tree level, except now that the space-time size is reduced by a factor $1/(G+1)$.

In interpreting the causal time-delay of open string scattering at tree level [10], key observations have been that the extension of the open string along the longitudinal direction grows linearly with energy, $L \sim p_0 \alpha'$, and that this leads to causal time-delay whose magnitude is proportional to L and hence to p_0 .[§] From this point of view, the fact that saddle-point trajectories of the open string are all universal enables us to understand intuitively reduction of the causal time-delay (3.5) at higher orders. At G -th order in perturbation theory, size of each open string along the longitudinal direction (scattering axis) is effectively scaled down by $1/(G+1)$ compared to that at tree level. Consequently, when scattered off each other, the open strings at G -th order would display the causal time-delay only by $1/(G+1)$ times that at tree level. This intuitive explanation is in complete agreement with our earlier result (3.5). Stated differently, as the incoming strings are folded or wound around $(G+1)$ times, their tension is effectively increased by a factor $(G+1)$ as in (2.14).[¶] Correspondingly, the causal time-delay is reduced

[‡]To be more precise, there exist the cases in which we have only one intermediate string. They occur when $L_i + L_j \neq 0 \pmod{G+1}$ in (2.7). However, they do not make a significant difference in later discussions. See also the footnote below Eq. (2.7).

[§]Here, one assumes implicitly that the longitudinal direction of a string could be identified definitely. However, it is not evident that this is always possible especially at high energy where a string would oscillate frequently. Presumably our scattering should be argued in the infinite momentum frame which is the most natural in high-energy scattering.

[¶]As the string is folded over, one might have anticipated that the tension is reduced instead of being increased.

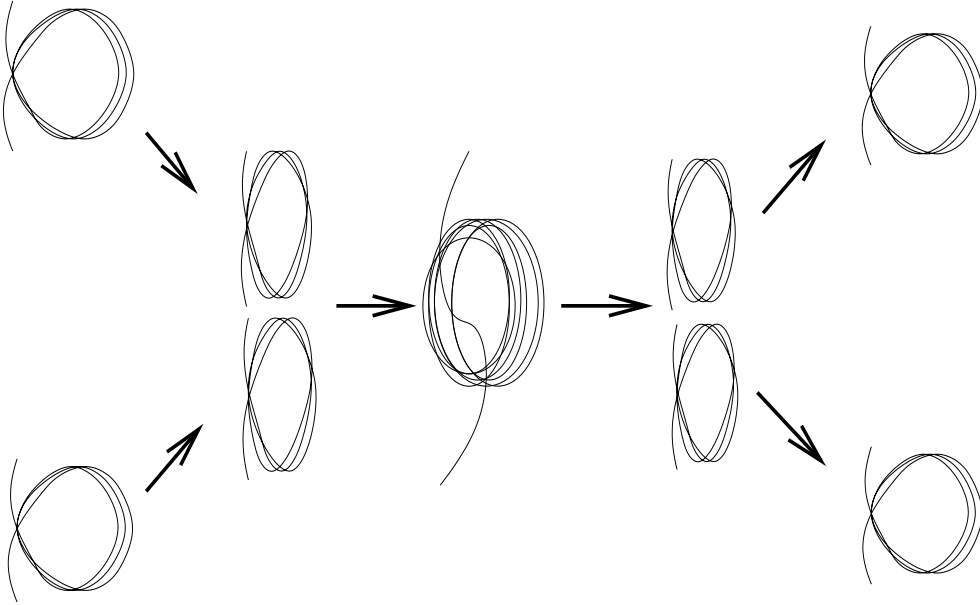


Figure 1: Space-time picture of the high-energy open string scattering at G -th order in perturbation theory.

as given in (3.5). Turning around the argument, from high-energy scattering in perturbation theory, we have confirmed that the causal time-delay exhibited therein is consistent with the interpretation that the longitudinal size of the string grows linearly with energy, $L \sim p_0 \alpha'$ (see (4.1)). This seems also consistent with the space-time uncertainty principle [17, 18], on which we will discuss more later.

In concluding this section, we summarize the space-time trajectory of high-energy open string scattering [21] as follows: according to (4.1) and the reflection principle, each of the two incoming open strings is in $(G + 1)$ -times multiply wound or folded configuration. As the two open string endpoints interact, at the scattering point, the wound or folded strings rearrange themselves into a configuration consisting of at most two short open strings and $\sim G/2$ many singly wound, short closed strings. Characteristic size of these short strings is that of incoming open strings, viz. $1/(G + 1)$ -times smaller than the size at tree level. After interaction, the little strings rejoin and split off into two outgoing open strings, each of them are again $(G + 1)$ -times multiply wound or folded. In general, these outgoing open strings would have different lengths from the incoming ones. A cartoon view of the space-time trajectory of the scattering process is depicted in Figure 1.

However, this argument applies only to nonrelativistic string such as polymer. For relativistic string, the tension remains always equal to the mass density.

5 Discussions

5.1 Comparison with Space/Time Noncommutative Field Theory

Let us compare our results with those expected in the space/time noncommutative field theory. As shown in [10], at tree level, the noncommutative field theory exhibits both advanced and retarded scattering. Even though the theory is generically non-unitary [11], one might put it aside and inquire of acausality at higher orders in perturbation theory: are the advanced/retarded effects present at higher orders in perturbation theory and, if so, are the effects increased or reduced as compared to those at tree level? It is well known that, in the maximal noncommutativity limit, $\theta^{\mu\nu} \rightarrow \infty$, the scattering amplitudes of noncommutative field theory are dominated by planar loop graphs. This is so because all nonplanar loop graphs are completely suppressed due to the maximal noncommutativity. In planar graphs, there is no θ -dependence apart from the overall Moyal phase associated with external momenta. These overall Moyal phase is the same for all planar graphs no matter how higher order they come from in perturbation theory. As such, at any order in perturbation theory, one does not expect any reduction of the retarded or advanced scattering behavior in the maximally noncommutative field theory. This should be contrasted to the fact that, in open string theory, universal structure of the saddle points has played an essential role in the reduction of the causal time delay. Indeed, exponential fall off and universal structure of the high-energy scattering amplitude (2.10) are characteristic features of open string theory, which are not present in quantum field theories with a finite number of field degrees of freedom.

5.2 Relation to Space-time Uncertainty Relation

Let us make comparison of what we have found with the space-time uncertainty relation: [17, 18]

$$\Delta X \Delta T \gtrsim \alpha'. \quad (5.1)$$

First it is worth noting that in our case this inequality is far from being saturated at lower order in perturbation theory. One might naively estimate that, in the present context,

$$\Delta X \sim \frac{p_0 \alpha'}{G+1} \quad \text{and} \quad \Delta T \sim \frac{1}{E} \sim \frac{1}{p_0}, \quad (5.2)$$

such that the space-time uncertainty relation (5.1) is not obeyed. However, ΔT should include not only the quantum fluctuation uncertainty but also the causal time-delay effect and, at high energy $p_0^2 \alpha' \gg 1$, the latter is the dominant effect:

$$\Delta T \sim \frac{p_0 \alpha'}{G+1}. \quad (5.3)$$

Thus, at a fixed order in perturbation theory, the left-hand side of (5.1) is much bigger than α' .^{||} Even including time-delay effect, the fact that both ΔX and ΔT become smaller at higher order as estimated above implies that the space-time uncertainty relation (5.1) would be saturated at the order $G_{\max} \sim \sqrt{p_0^2 \alpha'}$. Does this mean that the space-time uncertainty relation is violated beyond that order in perturbation theory? We believe not so.

A possible resolution is that the causal time-delay may be enhanced further at higher-order in perturbation theory. Indeed, in case of high-energy *closed* string scattering, via Borel-resummation of fixed order saddle-point results [23], it has been already argued that high-energy asymptotic behavior could be significantly modified at nonperturbative level. After Borel-resummation, the scattering amplitude turns out to behaves as

$$\mathcal{A}_{resum}(s) \sim e^{-\sqrt{s}} \quad (5.4)$$

in the kinematical range $\log(1/g^2) \ll s \ll 1/g^{4/3}$ and exhibits much bigger amplitude than any fixed-order perturbative behavior, $e^{-s/(G+1)}$. Thus, once Borel-resummed, it might be that causal time-delay is considerably different from the fixed-order estimate (5.3). It would even be the case that the inequality in the space-time uncertainty relation (5.1) is saturated nonperturbatively in the high-energy regime, where the symmetry of the string theory is also believed to be enhanced [22] enormously. In [18], the time-delay is indeed discussed based on (5.4). However, it should be noted that (5.4) was derived by the Borel resummation and as such it is one of possible guesses for nonperturbative high-energy amplitude. On the other hand, our time-delay is based on the amplitude at the saddle point which is certainly valid in the high-energy scattering with a fixed moderate angle. In this sense, we believe that although our time-delay is theoretical in itself, its reduced nature is important and that it indeed suggests the nonperturbative saturation of the space-time uncertainty relation. This is because if it were to grow with the order of the perturbation theory, it is impossible to expect such saturation. One can also find another importance of our time-delay in the following heuristic argument: suppose it is also possible to take advantage of the saddle point method in the summation over G as discussed in [19], we find that the dominant order is given by $G \sim \sqrt{-\alpha' s / \log g}$, and then we get $\Delta X \sim \Delta T \sim \sqrt{\alpha'}$ up to some logarithmic corrections by substituting this into our time-delay. Therefore, this argument using our time-delay also implies that the space-time uncertainty relation is almost saturated at nonperturbative level.

Another possible resolution is that yet another new sort of nonperturbative effects begin to appear at the order G_{\max} . For example, as the strings are boosted to infinite momenta, $p_0 \sim p_{\parallel} \sim N/R$ (R is an appropriate infrared cutoff scale) and hence $G_{\max} \sim N$. This is the same order as the light-cone momenta p_+ and hence the width of the light-cone string diagram.

^{||}We would like to thank T. Yoneya for pointing out this effect.

We have seen, in previous sections, that the saddle-point trajectory is such that the string folds or winds around G -times. The fact that there is a maximal folding or winding and that each elementary folding or winding carries $\mathcal{O}(1)$ unit of p_+ momentum indicate that sub-structure of the open string begins to show up. Indeed, the effect is strikingly reminiscent of the matrix string theory, where a generic string configuration is built out of minimal length strings, each carrying precisely one unit of p_+ . Using matrix string theory, it has even been found [24] that D-brane pair-production becomes an important effect when the fragmented strings produced during the high-energy scattering are all of minimal length, viz. $G \sim G_{\text{max}}$. Whether this striking similarity is a mere coincidence or not deserves further study, especially, in light of the fact that the D-brane pair production at high-energy scattering has been found for closed strings [24] and hence cannot be a feature unique to open strings only.

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